Sequences and Series 3

1. An investor intends to invest $\$ W$ in a finance company. The dividend of $d\%$ per year that he receives is reinvested in the company. Write the total of his investment that accumulates at the end of the first year, second year and third year. Deduce the total investment in $, that accumulates at the end of year n, including the dividend for the nth year.

2. Find the sum of $\frac{1}{1\times 5}+\frac{1}{3\times 7}+\frac{1}{5\times 9}+\frac{1}{7\times 11}+\cdots+\frac{1}{(2n-1)(2n+3)}$

3. Given $f(r)=r!$, find $f(r+1)-f(r)$. Hence, find the sum of the first 2n terms of the series
$1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots$

4. Given $u_n=\frac{1}{\sqrt{2n-1}}-\frac{1}{\sqrt{2n+1}}$, express $\sum_{n=25}^{N} u_n$ in terms of N. Deduce the value of $\sum_{n=25}^{\infty} u_n$.

5. If $5^2\times 5^4\times 5^6\times \ldots\times 5^{2p}=(0.04)^{-28}$, determine the value of p, where $p>0$.

6. **(i)** Prove that the sum of the first n terms of an arithmetic series:

$$a+(a+d)+(a+2d)+\cdots \text{ is } \frac{n}{2}[2a+(n-1)d].$$

(ii) Show that $1^2-2^2+3^2-4^2+\cdots+(2n-1)^2-(2n)^2=-n(2n+1)$
Deduce the sum of the following series:
(a) $1^2-2^2+3^2-4^2+\cdots+(2n-1)^2-(2n)^2+(2n+1)^2$,
(b) $21^2-22^2+23^2-24^2+\cdots+39^2-40^2$.

7. Given that the first term of a Geometric Progression is 8 and the sum of the first three terms is 104, find the possible values of the common ratio. In each case, write down the corresponding first three terms of the sequence.

8. If the sum of the 1st and 4th term of a Geometric Progression is 52, and the sum of the 2nd and 5th terms of the same Geometric Progression is -156, find the sum of the first eight terms.

9. Express $\frac{1}{(1-r)r}$ in partial fractions. Hence, find $\sum_{r=2}^{n}\frac{3}{(1-r)r}$.

10. The rth term, u_r, of an infinite series is given by $u_r=\left(\frac{1}{3}\right)^{2r+1}+\left(\frac{1}{3}\right)^{2r-1}$.

(a) Express u_r in the form $\frac{A}{3^{2r+1}}$, where A is a constant.

(b) Find the sum of the first n terms of the sequence, and deduce the sum of the infinite series.